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Credit Risk and Credit Derivatives in Banking

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Abstract

Using the industrial economics approach to the microeconomics of banking we analyze a large bank under credit risk. Our aim is to study how a risky loan portfolio affects optimal bank behavior in the loan and deposit markets, when credit derivatives to hedge credit risk are available. We examine hedging without and with basis risk. In the absence of basis risk the usual separation result is confirmed. In case of basis risk, however, we find a weaker notion of separation.

Keywords: credit risk, credit derivatives, banking firm, risk aversion

JEL classification: G21

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Credit Risk and Credit Derivatives in Banking

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Using the industrial economics approach to the microeconomics of banking we analyze a large bank under credit risk. Our aim is to study how a risky loan portfolio affects optimal bank behavior in the loan and deposit markets, when credit derivatives to hedge credit risk are available. We examine hedging without and with basis risk. In the absence of basis risk the usual separation result is confirmed. In case of basis risk, however, we find a weaker notion of separation.

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1 Introduction

Credit risk is one of the oldest and most important forms of risk faced by banks as financial intermediaries. The risk of borrower default — on interest and/or principal — carries the potential of wiping out enough of a bank's capital to force it into bankruptcy. Managing this kind of risk through selecting and monitoring borrowers and through creating a diversified loan portfolio has always been one of the predominant challenges in running a bank.

Since the 1980s a number of new risk sharing markets and financial instruments have become available which make credit risk more manageable (see Neal, 1996, and Bank for International Settlements, 2001). Banks can pool assets with credit risk and sell parts of the pool. This asset securitization or creation of asset backed securities has seen considerable growth in areas such as home mortgages or automobile loans, where underlying loan contracts and payment schedules are fairly standardized and risk characteristics are similar. Loan sales play a role, e.g., in takeover financing, where a bank originates a loan and sells it in smaller shares to other banks. More recently, credit derivatives such as credit swaps, credit options, and credit-linked notes have gained importance as instruments to manage risk in situations, where the diversity of loan types and credit risks makes it difficult to securitize loans or sell them individually.

In the sequel we will use the term credit derivatives both for securities originating from loan securitization and for more advanced instruments such as credit options. Our objective is to examine how the possibility to sell part or all of a bank's uncertain loan portfolio at a deterministic price affects bank behavior in deposit and loan markets.

The framework we use for our analysis is sometimes called the industrial organization approach to the microeconomics of banking (for a brief survey see Freixas and Rochet, 1997, chapt. 3). It is focused on the bank's role as intermediary, but abstracts from informational aspects of banking — adverse selection and moral hazard — which have dominated banking theory throughout the last two decades. We consider the potential of the industrial organization approach to analyze banking under a variety of market structures ranging from perfect competition to monopoly sufficiently important to justify the use of this approach.[1] To our knowledge, Wong (1997) was the first author to add aspects of uncertainty and risk aversion to the industrial organization approach to the bank. We supplement Wong's analysis of credit risk by adding a hedging instrument which may or may not carry basis risk. Since the seminal work of Froot et al. (1993) hedging is known to contribute to a firm's market value. In our treatment of deposits, we deviate from Wong (1997) by assuming a deterministic deposit rate and modelling an explicit deposit taking decision of the bank.

The plan of the paper is as follows. In section 2 we present the model of a large banking firm under credit risk, when a credit derivative is available. Section 3 examines loan, deposit and hedging decisions for a credit derivative without basis risk. Section 4 adds basis risk to our analysis. Section 5 concludes the paper.

2 The model

Consider a large banking firm in a one–period framework. The bank is a classical intermediary, taking deposits D and making loans L. By "large" we mean that the bank faces a downward sloping inverse demand $r_L(L)$ for loans with r_L denoting the interest rate on loans and an upward sloping supply $r_D(D)$ of deposits with r_D denoting the interest rate on deposits. Both demand for loans and supply of deposits are assumed to be deterministic. The case of perfect competition can easily be considered in this framework. However, by making the assumption of a single large bank we deliberately neglect the strategic interactions among banks under an oligopolistic market structure. An analysis of a banking duopoly with credit uncertainty and hedging will be reserved for future research.

The bank is required by regulation to hold a portion $\alpha \in (0, 1)$ of its deposits

[1] For an analysis combining aspects of market structure and asymmetric information see e.g. Gehrig and Stenbacka (2001).

as non–interest bearing reserves. It faces operational costs $C(D,L)$ with strictly positive marginal costs C'_D and C'_L. Assumptions on second derivatives of the cost function will be discussed later when they are needed to derive results on optimal behavior. Equity capital, K, of the bank is taken as given. The balance sheet constraint of the bank can be written as

$$M = K + (1-\alpha)D - L \tag{1}$$

The bank's interbank market position, M, can take a positive or a negative value, implying lending or borrowing in the interbank market at an interest rate r assumed to be deterministic and given. To motivate the existence of an interbank money market, imagine our bank being one of a large number of local monopolists or a central bank providing liquidity to the banking system at a rate r.

The bank faces credit risk in the sense that a stochastic portion $\tilde{\theta}$ of the loan volume will turn out to be non–performing. The random variable $\tilde{\theta}$ follows a distribution function defined on the interval $[0,1]$. A loan is defined as non–performing, if the borrower does not pay interest in the period under consideration, i.e., we do not assume that the loan has to be written off completely, leading also to a loss on the principal. Extending the model to the case of write–offs poses no difficulty, but offers no additional insights and leads to some more complicated formal expressions.

Given credit uncertainty, the random profit of the bank is defined as

$$\tilde{\Pi} = (1-\tilde{\theta})r_L(L)L + rM - r_D(D)D - C(D,L). \tag{2}$$

$\tilde{\Pi}$ consists of the uncertain interest earned on loans plus the positive or negative interest on the interbank position minus interest paid on deposits and operational costs.

As noted in the introduction, financial markets today offer new financial instruments which alleviate risk management. The creation of instruments to manage credit risk may be one of the most important steps towards complete risk sharing markets. In the sequel we analyze the impact of credit derivatives on a bank's optimal deposit and loan decisions and its risk management. We assume the existence of a market for credit derivatives. As noted before, we neglect the huge variety of real–world forms of credit derivatives and model a most simple hedge instrument which corresponds to a total return swap. The credit derivative offers an exchange of an uncertain future cash flow against a certain cash flow. By selling a volume H of the derivative the bank agrees to exchange a stochastic claim $H\tilde{\theta}$ against a deterministic claim $H\bar{\theta}$ at the end of the period. $\bar{\theta}$ is the forward rate for one unit of credit risk. Seen from the beginning of the period, hedging therefore contributes $H(\tilde{\theta}-\bar{\theta})$ to the bank's profit. In this section we assume a perfect negative correlation between credit risk exposure and the gain or loss $H(\tilde{\theta}-\bar{\theta})$ from hedging. This absence of basis risk assures that credit risk can completely be traded away.

Substituting the balance constraint (1) for M in (2) and taking account of hedging leads to a modified profit function of the bank:

$$\tilde{\Pi} = ((1-\tilde{\theta})r_L(L) - r)L + ((1-\alpha)r - r_D(D))D + rK - C(D,L) + H(\tilde{\theta} - \bar{\theta}). \quad (3)$$

In (3) we have used the fact that the bank's balance sheet constraint has not changed due to participation in the market for derivatives since derivatives contracts only define payments to be made at the end of the period. Further, notice that the volume H of contracts sold is not constrained. This means for $H > 0$ the bank sells credit derivatives, whereas in the case of $H < 0$ it is a buyer of the hedging instrument.

The bank's owners or managers maximize a von Neumann–Morgenstern utility function $U(\Pi), U' > 0$, which exhibits risk aversion, i.e., $U'' < 0$ (for a theoretical basis of the assumption of risk aversion see Froot and Stein, 1998, and — in the framework of the industrial organization approach to banking — Pausch and Welzel, 2002). This leads to the expected utility maximization problem

$$\max_{D,L,H} \mathrm{E}[U(\tilde{\Pi})] \quad (4)$$

where $\tilde{\Pi}$ is defined by (3) above.

3 Hedging without basis risk

The first order necessary conditions for (4) are given by

$$\begin{aligned} \mathrm{E}[U'(\tilde{\Pi}^*)(1-\alpha)r - r_D(D^*) - r'_D(D^*)D^* - C'_D(D^*,L^*)] &= 0 \quad (5)\\ \mathrm{E}\left[U'(\tilde{\Pi}^*)\left((1-\tilde{\theta})(r_L(L^*) + r'_L(L^*)L^*) - r - C'_L(D^*,L^*)\ \right)\right] &= 0 \quad (6)\\ \mathrm{E}[U'(\tilde{\Pi}^*)(\tilde{\theta} - \bar{\theta})] &= 0 \quad (7) \end{aligned}$$

Examination of (5), (6) and (7) leads to the following

Proposition 1 *Given a credit derivative with perfect negative correlation with the bank's exposure to credit risk, (a) the bank can separate its decision on risk management from its decisions on deposit and loan volumes, (b) the bank fully hedges its credit risk exposure, if the hedge instrument is unbiased.*

Proof (a) Substituting $\mathrm{E}[U'(\tilde{\Pi}^*)\bar{\theta}]$ for $\mathrm{E}[U'(\tilde{\Pi}^*)\tilde{\theta}]$ from (7) in (5) and (6) yields two deterministic equations in D and L which can be solved for the optimal values D^* and L^*:

$$\begin{aligned} (1-\alpha)r - r_D(D^*) - r'_D(D^*)D^* - C'_D(D^*,L^*) &= 0 \quad (8)\\ (1-\bar{\theta})\left(r_L(L^*) - r'_L(L^*)L^*\right) - r - C'_L(D^*,L^*) &= 0 \quad (9) \end{aligned}$$

(b) If the derivative market is unbiased, i.e., $\mathrm{E}(\tilde{\theta}) = \bar{\theta}$, $\mathrm{Cov}[U'(\tilde{\Pi}^*), \tilde{\theta}] = 0$, which implies a deterministic Π^*. This in turn implies that the bank has no exposure to risk, i.e. there is a full hedge $H^* = r_L L^*$. q.e.d.

Part (a) of the proposition is an example for the well–known separation property in the presence of a hedging instrument without basis risk. As a consequence the bank will choose the same volumes of deposits and loans as in the case of a deterministic rate $\bar{\theta}$ (certainty equivalence).

Introducing the elasticity of supply of deposits $\epsilon_D = (dD/dr_D)(r_D/D)$ and the elasticity of loan demand, $\epsilon_L = -(dL/dr_L)(r_L/L)$, (8) and (9) can be re–written as

$$\frac{(1-\alpha)r - r_D(D^*) - C'_D(D^*, L^*)}{r_D(D^*)} = \frac{1}{\epsilon_D^*} \tag{10}$$

$$\frac{(1-\bar{\theta})r_L(L^*) - r - C'_L(D^*, L^*)}{(1-\bar{\theta})r_L(L^*)} = \frac{1}{\epsilon_L^*} \tag{11}$$

These are the familiar equalities between a Lerner index (price minus marginal cost divided by price) and an inverse elasticity adapted to the case of banking (cf. Freixas and Rochet, 1997, p. 58). Greater market power in the market for deposits, i.e., a smaller value of ϵ_D, implies a higher Lerner index and a higher intermediation margin. For $\epsilon_D \to \infty$ the model leads to the limiting case of perfect competition in the deposits market where the interest margin, $(1-\alpha)r - r_D(D)$, just equals marginal operating costs C'_D. This holds analogously for the loan market.

4 Hedging with basis risk

In the previous section we considered a market for credit derivatives which permitted the bank to perfectly avoid exposure to risk. In reality selling all credit risk may not be possible. We refer to the non–tradeable risk as basis risk. The most important causes of basis risk discussed in the literature are differences in the maturities of the hedging instrument and the bank's risky position, and differences in the stochastic properties between the underlying of the hedging instrument and the risk the bank faces. In the case of credit risk the first problem arises when the derivatives contract matures at an earlier date than the underlying loan contract. As an example for the second cause of basis risk consider the case of an underlying of the credit derivative which is not perfectly correlated with the credit risk. The latter aspect appears to be of minor importance since credit derivatives are usually traded over the counter which should imply that the contracting parties search for an underlying with a very high correlation to the risk at hand. In addition, we can think of the risk of giving loans in perfect analogy to the risk of holding shares. Part of the risk is systematic (market risk), part of it is unsystematic (idiosyncratic risk) (cf. Diamond, 1984). In the case of a loan, systematic risk is primarily driven by macroeconomic conditions, whereas unsystematic risk is caused by characteristics of the debtor and his project. Systematic risk is tradeable. It contributes most of the total risk of a loan (cf. Wilson, 1998). Unsystematic risk should be avoided by the bank itself through creating a diversified loan portfolio. However, banks may find it difficult to fully diversify this idiosyncratic risk, because they face institutional constraints, such as credit unions in the U.S. or cooperative banks and savings banks in Germany, or are focused on specific sectors of the economy. However, this non–diversifiable unsystematic risk is also non–tradeable due to the information problems attached to the loan contract: A potential buyer is at an informational disadvantage compared to the bank willing to sell. We conclude from this discussion that non–tradeable credit risk may exist and should therefore be analyzed as basis risk in the framework of our model.

Consider a market for total return swaps as described in the previous section. To model basis risk we introduce the following modification: The market uses no longer the share of non–performing loans $\tilde{\theta}$, but a share $\tilde{g}$ as underlying of the derivatives contract. $\tilde{g}$ can be interpreted as the share of loans non–performing due to systematic risk. From this definition it is apparent that the two risks are not necessarily independent. We assume regression dependence between the two random variables (cf. Benninga et al., 1984), i.e.,

$$\tilde{\theta} = b + \beta\tilde{g} + \tilde{s} \tag{12}$$

where $b \geq 0$, $\beta > 0$ and $\tilde{s}$ is a zero mean noise term stochastically independent from $\tilde{g}$. For each unit of the credit derivative sold the bank receives a deterministic payment $\bar{g}$ in exchange for the stochastic amount $\tilde{g}$.

We assume unbiasedness of the derivatives market, i.e., $\mathrm{E}(\tilde{g}) = \bar{g}$, with $\bar{g}$ denoting the market price of the underlying chosen by the contracting parties. This implies $\bar{\theta} = \beta\bar{g}$, where we assume $b = 0$ without loss of generality.

The bank's profit can now be re–written as

$$\tilde{\Pi} = \Big((1-\tilde{\theta})r_L(L) - r\Big)\,L + ((1-\alpha)r - r_D(D))\,D + rK - C(D,L) + H(\tilde{g}-\bar{g}). \tag{13}$$

Maximizing (4), where $\tilde{\Pi}$ is now given by equation (13), yields (5) and (6) as in the case without basis risk. Condition (7) for the optimal hedge volume, however, is replaced by

$$\mathrm{E}\Big[U'(\tilde{\Pi}^*)(\tilde{g}-\bar{g})\Big] = 0 \tag{14}$$

Inspection of the first order conditions leads us to

Proposition 2 *(a) In the presence of basis risk the bank hedges a portion β of the uncertain interest payment $r_L L^*$ (beta–hedge rule). (b) The usual separation property no longer exists. Instead, a weaker notion of separation holds. (c) In the absence of economies or diseconomies of scope, the optimal volume of deposits D^* can be determined as in the case of certainty.*

Proof (a) Unbiasedness of the derivatives market implies that (14) can be written as $\mathrm{Cov}[U'(\tilde{\Pi}^*), \tilde{g}] = 0$. Replacing $\tilde{\Pi}^*$ by (13) and using (12) yields

$$\mathrm{Cov}\left[U'\left(\tilde{s} r_L L^* - \tilde{g}(\beta r_L L^* - H^*) + \mathrm{const.}\right), \tilde{g}\right] = 0 \tag{15}$$

Due to the stochastic independence of $\tilde{s}$ and $\tilde{g}$ this can only be true, if

$$\beta = \frac{H^*}{r_L L^*} \tag{16}$$

(b) Inserting (12) and the optimal hedge rule (16) into the first order condition (6) for loans shows that L^* still depends on probabilities and risk preferences, even if D^* were known. This in turn implies from (5) that D^* also cannot be determined without knowledge of probabilities and risk preferences. More than market data is required to decide the optimal loan and deposit volumes, which prevents the traditional notion of separation of production and risk management.

Notice, however, that the optimal hedge rule derived holds for any pair (D, L). We can therefore imagine a bank choosing loan and deposit volumes randomly and still minimizing its risk exposure by applying the beta–hedge. While the bank may find it impossible to determine the optimal values of D^* and L^* in the presence of basis risk, it can still separate its hedging decision from its production decisions. We call this a weak notion of separation.

(c) Inspection of (5) shows that for $C_{DL} = 0$ (neither economies nor diseconomies of scope) D^* can be determined on the basis of market data alone, i.e., without knowledge of probabilities, risk preferences, or the bank's hedging decision. q.e.d.

5 Conclusion

Using the industrial organization approach to the microeconomics of banking, we analyzed the implications of credit risk and credit derivatives without and with basis risk for optimal bank behavior under risk aversion. Under perfect correlation between credit risk and derivative, the familiar separation property was confirmed for the banking firm. A full hedge turned out to be optimal, if the market for derivatives is unbiased. The usual separation result no longer holds in the presence of basis risk, i.e., optimal loan and deposit volumes depend on risk preferences, expectations etc. However, the beta–hedge rule derived for this case is optimal irrespectively of the loan and deposit volumes chosen. In this sense, there is still a separation of production decisions and risk management.

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