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Education, Research and the Impact of Tuition Fees
A Simple Model of the University

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Abstract. The present paper analyses the behaviour of a university within a neoclassical equilibrium framework. Demand for enrolments is traced back to the decision of potential students which aim at maximizing expected lifetime income. Here, the key factors are the students' preferences and abilities, the quality of education offered by the university and several external determinants like, e.g., tuition fees and differences in income between graduates and non-graduates. In turn, the behaviour of the university in terms of educational efforts and the strength of academic standards depends on the demand for enrolments, on financial resources available and on the specific objectives pursued by the university. The main emphasis is on the implications of different funding mechanism (governmental grants vs. tuition fees) in combination with different objectives pursued (maximizing enrolments vs. maximizing prestige via research output). It is shown that for given financial resources a university that aims at maximizing prestige always provides only a lower quality of education for a smaller number of students compared to a university that aims at maximizing enrolments. Moreover, the effects caused by changes in governmental grants or tuition fees are quite different depending on the university's objectives. Yet, there is also one common feature: Irrespective of which utility function is maximized, partially substituting governmental grants by tuition fees would change neither educational efforts nor academic standards, but it would inevitably lead to decreasing enrolments. As a positive side-effect, however, the average ability of the remaining population of students would increase.

Keywords: Educational Economics, University, Tuition Fees.

JEL Classification: A2.

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1. Introduction[1]

Although economists have traditionally paid only minor attention to non-profit organizations, a lot of research has been done on the economics of universities since Veblen's pioneering work „The Higher Learning in America“ (1918). Most of this research, however, is dominated by human capital theory and concentrates on the determination of returns on investments in higher education. Compared to this, only little work has been done on the institutional behaviour of universities and its determinants within the market context.[2] Moreover, even in this subgroup of literature, most of the work is rather descriptive and lacks of formal strength. There are surprisingly little attempts to rigorously analyse the behaviour of universities within a neoclassical equilibrium framework that explicitly accounts for utility maximization by the individuals involved. According to a recent survey by Raines/Leathers (2003, pp. 186) the most comprehensive and sophisticated approach available in the literature is still the model presented by Garvin (1980, pp. 21). However, even this model yields only limited insights into the issue at hand since it concentrates on the supply side and pays no attention to the question, how the demand function for enrolments is determined. Moreover, Garvin refrains from any comparative statics and arrives only at some fairly general optimality conditions which just reiterate the well known requirement to equate marginal costs and benefits of all decisions taken by the university. Consequently, the model does not provide any clue to important policy issues like, e.g., the reaction of the university and the accompanying change in enrolments if government decides to partially substitute governmental grants paid to the university by increasing tuition fees.

The present paper tries to shed some light on issues like this using an equilibrium model that accounts for the maximization behaviour of all individuals involved. In short, demand for enrolments is traced back to the decision of (potential) students which aim at maximizing expected lifetime income. The key factors of this decision process are the students' preferences and abilities, the quality of education offered by the university and several external determinants. In turn, the behaviour of the university in terms of educational efforts and the strength of academic standards depends on the demand for enrolments, on the amount of financial resources available and on the specific objec-

[1] The author gratefully acknowledges helpful comments by Albrecht Bossert and Jürgen Dietz (University of Augsburg, Department of Economics). Of course, the usual disclaimer applies.

[2] See, e.g., the extensive collection of surveys on the economics of universities edited by Hoenack and Collins (1990). For more recent contributions see Johnes (2000) and Raines/Leathers (2003).

tives pursued by the university. A detailed description of the model is given in Section 2. In Section 3, the equilibrium conditions are derived and Section 4 provides a throughout comparative statics analysis. Finally, Section 5 closes the paper with a short summary of the main findings and some remarks on promising routes for future research.

Before proceeding, however, on important qualification should be noted. The model presented below follows the tradition of Garvin (1980) and attempts an entirely positive economic analysis. Hence, it does not ask, how a university should behave from the viewpoint of social welfare, but it asks how universities actually behave. Within this context, the main emphasis is on the implications of different funding mechanisms (governmental grants vs. tuition fees) in combination with different objectives pursued by the university (maximizing enrolments vs. maximizing prestige via research output).

2. The model

2.1 Demand side

In order to keep the model as simple as possible and to avoid complications associated with competition between different universities, it is assumed that there exists only one university in the geographical area under consideration and potential students (i.e., individuals who just have accomplished university entrance qualification) are completely immobile.[3] Potential students only have to decide whether to enrol at university or to start working within the non-academic sector. This decision, however, is subject to uncertainty since students once enrolled cannot be sure whether or not they finally will become graduated. In general, it seems sensible to assume that the probability to become graduated depends on an individual's innate ability in terms of coping with the requirements of academic education as well as on the characteristics of education supplied by the university. To capture this decision problem, indicate potential students by j=1,2,…,n, and let us assume that the innate ability of individual j is given by $\alpha^j \in [0,1]$. Moreover, denote the strength of the university's academic standards by $q \in [0,1]$ and its educational efforts, measured as the ratio between financial resources devoted to instruction and the number of students enrolled, by $e > 0$. Now, let us combine these factors in order to build a "success function" which describes the probability that individual j will become graduated if she chooses to enrol at university:

[3] Neglecting competition is almost standard in economic models of the behaviour of universities as complained by Rothschild/White (1993, p.11). Nevertheless, as pointed out by Garvin (1980, p.9), the assumption of immobility of potential students and its implications concerning limited competition between universities seems to be more or less realistic except for universities with a very high reputation for excellence which attract students from a larger geographic market.

[1] $w^j = \alpha^j \cdot w(e,q)$.

Hence, individual j's probability to become graduated if she chooses to study is c.p. the higher, the higher her own ability α^j is. Moreover, the function w(e,q) is assumed to be continuous, twice differentiable and to satisfy the following properties for any level of e>0 and $q \in [0,1]$:

$$w(e,q) \in [0,1],\ \lim_{e \to 0} w(e,q) = \lim_{q \to 1} w(e,q) = 0,\ w'_e > 0,\ w''_{ee} < 0,\ w'_q < 0,\ w''_{qq} < 0 \text{ and } w''_{qe} \geq 0.$$

Since w(e,q) is the most important driving force within the present model, the above assumptions should carefully be recognized. Due to $w'_e > 0$ and $w'_q < 0$ the probability to become graduated is c.p. the higher, the higher are the university's educational efforts and the lower are its academic standards. The second derivative $w''_{ee} < 0$ indicates decreasing marginal "productivity" of educational efforts. The second derivative $w''_{qq} < 0$ indicates an increasing marginal reduction in the probability to become graduated if academic standards are tightened. Moreover, if educational efforts approaches zero the probability to become graduated also tends to zero irrespective of academic standards. Similarly, for any level of educational efforts the probability to become graduated tends to zero if academic standards approaches an upper bound which is set at q=1.[4] Finally, the assumed sign of the cross derivative w''_{qe} indicates that the reduction in the probability to become graduated caused by a marginal strengthening of academic standards is decreasing or at least constant if educational efforts are increased.

Next, let us assume that the discounted lifetime income of an individual who decides not to study at all is given by $\overline{R}$. Graduates of the university under consideration receive a discounted lifetime income of $(1+zq)(1-\gamma)\overline{R} - p$ with z>0, 0<γ<1 and p>0. The term (1+zq) reflects that higher academic standards c.p. lead to a higher future income of the university's graduates,[5] p represents a tuition fee charged by the university and the term $(1-\gamma)$ corrects for the opportunity cost of education in terms of forgone income during studying.[6] In contrast to graduates, drop-outs suffer from time and

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[4] Taken together, these two assumptions assure that the model always leads to an interior solution with e>0 and q<1.

[5] Note that the model does not distinguish whether the differences in income between graduates and non-graduates are due to the accumulation of human capital or just due to a pure signalling effect (on this topic see, e.g., Belfield, 2000, Chap. 2). Assuming for example that the income of a graduate of a university with "average" academic standards q=0.5 is twice as high as the income of a non-graduate would imply z=2. On empirical data on the correlation between academic standards of universities and the average starting salary of graduates are provided by Rothschild/White (1993).

[6] Assuming for example that the time spent by a student at university accounts for ten percent of her total working life would imply γ=0.1.

money vainly spent at university, return to working within the non-academic sector and end up with a discounted lifetime income of only $(1-\gamma)\overline{R}-p$. Hence, the present value of expected lifetime income of individual j if she chooses to enrol at university is given by $E(R^j)=w^j[(1+zq)(1-\gamma)\overline{R}]+(1-w^j)[(1-\gamma)\overline{R}]-p$. Inserting the success function [1] as introduced above and cancelling terms, this can be re-calculated as:

$$[2] \qquad E(R^j)=(1-\gamma)[1+zq\alpha^j w(e,q)]\overline{R}-p.$$

At this stage of reasoning, there is a strong temptation to assert that individual j will choose to enrol at university only if this leads to an expected lifetime income higher than the lifetime income of a worker within the non-academic sector, i.e., $E(R^j)>\overline{R}$. However, money is not everything and studying for some years probably might be more fun than immediately starting to work. In order to account for these additional “leisure benefits” of studying, the model assumes that individual j will choose to study if $E(R^j)+\beta>\overline{R}$ with $\beta>0$.[7] As a consequence, individual j will refrain from studying if her ability α^j falls short of a certain minimum level $\overline{\alpha}$ which can be calculated from solving $E(R^j)+\beta=\overline{R}$ for α^j:

$$[3] \qquad \overline{\alpha}(e,q)=\frac{\gamma\overline{R}+p-\beta}{(1-\gamma)zqw(e,q)\overline{R}}.$$

According to [3], the minimum ability $\overline{\alpha}$ is c.p. the higher, the higher are the opportunity cost of studying given by $\gamma\overline{R}$ and the higher are the direct costs given by the tuition fee p. The latter result reiterates the well known observation that tuition fees are not only a source of funding but also an economic mechanism of selection which can be used to keep less talented individuals from studying (see, e.g., Rothschild/White 1993).

Now let us assume that the different individuals’ abilities $\{\alpha^1,\alpha^2,\ldots,\alpha^n\}$ are *uniformly* distributed on the [0,1]-space. Under this condition, the total number of students attracted by the university, s(e,q), easily can be calculated from $s(e,q)=n[1-\overline{\alpha}(e,q)]$:

$$[4] \qquad s(e,q)=n\left[1-\frac{\gamma\overline{R}+p-\beta}{(1-\gamma)zqw(e,q)\overline{R}}\right]$$

The implications of this demand function for higher education should carefully be noted. As indicated by [4], the number of students attracted is c.p. the higher, the

7 Of course, in order to ensure an interior solution, it has to be assumed that leisure benefits in terms of monetary equivalents are smaller than the costs of studying at all, i.e. $\beta<\gamma\overline{R}+p$. Otherwise, studying would be attractive even for the least talented individual with $\alpha^j=0$.

smaller is the tuition fee p,[8] the smaller are opportunity costs of forgone income $\gamma\overline{R}$, the higher are leisure benefits β, the higher are the differences in income between graduates and non-graduates indicated by z, and the higher are the university's educational efforts e. In contrast, the impact of academic standards on the number of students attracted (i.e., the sign of s'_q) is ambiguous due to $w'_q < 0$:

$$[5] \qquad s'_q = \frac{n(\gamma\overline{R} + p - \beta)[w(e,q) + qw'_q]}{(1-\gamma)zq^2 w(e,q)^2 \overline{R}}.$$

The reason for this ambiguity is straight forward: On the one hand, for given ability α^j and given educational efforts e, an increase in q reduces the student's probability to become graduated as easily can be seen from $w'_q < 0$. On the other hand, however, an increase in q also leads to a higher future income of those students who finally become graduated. Taken together, these two opposite effects imply that for given educational efforts, increased enrolments must not necessarily come at the expense of reduced academic standards. Instead, starting with a sufficiently small q, the effect on the future income of graduates might dominate such that a further increase in q might lead to a higher number of students attracted.

Before proceeding to the supply side by describing the university's objectives and constraints it should be recognized that the above model also allows to draw a conclusion about the "quality" of the students attracted in the sense of their average ability. Since only those individuals with $\alpha^j \in [\overline{\alpha},1]$ will decide to study, their average ability $\tilde{\alpha}(e,q)$ can readily be calculated from $\tilde{\alpha}(e,q) = 0.5[1 + \overline{\alpha}(e,q)]$:

$$[6] \qquad \tilde{\alpha}(e,q) = \frac{1}{2}\left[1 + \frac{\gamma\overline{R} + p - \beta}{(1-\gamma)zqw(e,q)\overline{R}}\right].$$

Comparing [4] and [6] also reveals that an increasing number of students attracted always comes at the expense of a lower average ability.

2.2 Supply Side

Now, let us turn to the university's decision problem. Irrespective of the specific objectives pursued, the university inevitably has to satisfy the budget constraint B≥C where B indicates total financial resources available and C indicates total costs. Within the present model, the university is financed partially by governmental grants and partially by tuition fees, both fixed by the government. Denoting governmental grants per

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[8] For a survey of empirical studies on the impact of tuition fees on enrolments see, e.g, Heller (1997).

student by g>0[9] and the tuition by p>0, total financial resources available to the university are given by $B=(g+p)\cdot s(e,q)$. Total costs can be divided in two major categories:[10] Education costs and costs due to research activities. Accounting for education costs per student given by the level of educational efforts, e, and denoting costs due to research activities by r, leads to total costs of $C=e\cdot s(e,q)+r$.[11] As a consequence, the budget constraint of the university is given by:

$$[7] \qquad (g+p)\cdot s(e,q) \geq e\cdot s(e,q)+r\,.$$

It should be noted that this budget constraint implies cross-subsidization between tuition and research which is standard in the literature on the economics of university behaviour (see, e.g., James 1990): The university carries out a profitable activity that might yield only little utility per se (i.e., teaching students) in order to finance an other activity that increases utility but does not cover its own costs (i.e., research activities). Of course, the degree of direct utility derived from teaching students depends on the objectives pursued by the university. Determining the objectives of a university is made difficult by its internal organization: A university is no single actor but consists of different subgroups without a simple hierarchy − in particular administration and faculty − which might pursue different and perhaps conflicting goals (see, e.g. James 1990; Tuckman/Chung, 1990).[12] However, concerning the objectives pursued by a university at the *aggregate* level, there seems to be a widespread consensus among economists that two − partially interrelated − factors are of major importance: The quantity of students attracted and the degree of prestige obtained within the scientific community (see, e.g., Raines/Leathers 2003, James 1990, Garvin 1980).

The quantity of students attracted is important from the viewpoint of faculty and administration because expanded enrolments are likely to be accompanied by an increasing number of faculty positions and a higher budget to secure expanded facilities like,

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[9] It could be objected that governmental grants spent to universities are usually a *fixed* of amount of money independent from the number of students actually attracted. This view might be suitable in the short-run. In the long-run, however, it seems more sensible to assume that an increasing number of students attracted is accompanied by increasing governmental grants.

[10] For an exhaustive discussion of the different costs involved by operating a university see, e.g., Getz/Siegfried (1991) and Brinkman (1990).

[11] It should be noted that this cost function implies constant returns to scale in education. This simplification seems to be justified since empirical evidence indicates only modest economies of scale. For an overview on several cost studies see Brinkman (1990) and Hoenack (1990).

[12] In some countries like, e.g., Germany, administration tasks are executed by faculty members themselves on the base of a rotation system. This organization often leads to a somewhat amateurish management but it might help to overcome the problem of conflicting goals as mentioned above.

e.g, laboratories and libraries.[13] Moreover, expanded enrolments endow the university with a larger pool of students for the selection of research assistants and it generally enhances the university's survival as an organization.

Prestige is the second major component entering the university's utility function since it increases the faculty members' market values, generally improves their self-respect and makes research grants easier to obtain (e.g., Garvin 1980, p. 23). Prestige mainly can be traced back to the quality and quantity of research output "produced" by the university.[14] With respect to this, the model assumes that financial resources devoted to research activities can be used as a proxy for the quality and quantity of research output, as measured by, e.g., appropriate indexes of publication productivity.[15] The amount of financial resources available for research can easily be evaluated from the budget constraint: $r(e,q)=(g+p-e)s(e,q)$. Using [4], this can be re-calculated as:

$$[8] \qquad r(e,q)=(g+p-e)\left[1-\frac{\gamma\overline{R}+p-\beta}{(1-\gamma)zqw(e,q)\overline{R}}\right].$$

To sum up, the university under consideration may derive utility from the number of students attracted, s(e,q), as well as from the amount of financial resources available for research, r(e,q). This approach is sufficiently general to describe the behaviour of a large spectrum of different universities that assign different weights to these objectives. As emphasized by Garvin (1980, p.35), in practice some types of universities are primarily concerned with maximizing enrolments, whereas other types of universities primarily aim at maximizing prestige. In order to focus on the implications of these different kinds of behaviour, the following analysis assumes that the university under consideration solely aims at maximizing *either* enrolments *or* the research budget. Of course, this assumption is somewhat artificial since in reality most universities aim at maximizing a combination of these two elements. But looking at the extremes will facilitate a clear-cut analysis of the different objectives' implications.

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[13] With respect to this line of argumentation, the economics of universities is very similar to the economics of bureaucracies (see, e.g., Downs 1967, Niskanen 1971). However, as pointed out by Garvin (1980, p.38), just applying the bureaucratic standard approach of budget maximization to the behaviour of a university would imply an oversimplification.

[14] Other factors which add to prestige are particularly the quantity and quality of Ph.D. programs (see, e.g., Raines/Leather 2003, p.194). This issue, however, is beyond the scope of the present model.

[15] It should be emphasized that this assumption, which is standard in modelling the behaviour of universities, neglects the possibility of economies of scale in research activities as well as the possibility of economies of scope resulting from the joint production of tuition and research. On this issues see, e.g., Brinkman (1990, p.123) and Johnes (2000, p.142).

3. Equilibrium

First, consider a university that solely aims at maximizing enrolments. Under this assumption, due to $w'_e > 0$ the whole budget available will be allocated to educational efforts, i.e., $e^* = g + p$. Hence, the only remaining variable to be decided about are academic standards q. Maximizing $s(e,q)$ and accounting for $e^* = g + p$ leads to the following first order condition (for the second order condition see Appendix I):

[9] $\quad s'_q = 0$.

As can be calculated from [4], $s'_q = 0$ implies $w(e,q) + qw'_q = 0$. The interpretation of this condition is straight forward. Marginally increasing q induces two opposite effects: it reduces the students' probability to become graduated and it increases the future income of those who finally become graduated. Condition [9] requires to choose q* in such a way that these two effects are balanced in order to maximize the number of students attracted. However, one important caveat should be noted. Since s(e,q)>0 requires $w(e,q) > (\gamma\overline{R} + p - \beta)/(1-\gamma)zq\overline{R}$, as can be seen from [4], educational efforts and thereby financial resources available per student, g+p, have to exceed a certain lower bound $(g+p)^{min}$ in order to ensure an interior solution where the university is able to attract a positive number of students at all. This lower bound is c.p. the higher, the higher are opportunity costs of studying minus leisure benefits and the smaller are the differences in income between graduates and non-graduates.

Now, consider a university that solely aims at maximizing prestige via maximizing research output. From differentiating $r(e,q) = (g + p - e)s(e,q)$ with respect to e and q we obtain the following first order conditions (for the second order conditions see Appendix I):

[10] $\quad s'_q = 0$,

[11] $\quad (g + p - e)s'_e - s(e,q) = 0$.

Condition [10] again requires to fix academic standards in such a way that enrolments s(e,q) are maximized for any given level of educational efforts. Condition [11] reflects that a marginal increase in the money spent on educational efforts per student, de>0, has two opposite effects on the research budget: On the one hand, r(e,q) decreases by s(e,q)de since more money is needed to instruct those students already enrolled. On the other hand, the number of students enrolled will increase by $s'_e de$ which leads to an increase in r(e,q) by $(g + p - e)s'_e de$ because any new student attracted adds an amount of $(g + p - e)$ to the research budget. In order to maximize r(e,q) for any given level of academic standards, [11] requires to fix educational efforts in such a way as to balance these two effects. Finally, taken together [10] and [11] imply that e and q have to be combined in such a way that the effects of marginally varying both variables on the research budget are equalized.

The above results reiterate that a university can benefit from enlarged enrolments not only directly but also indirectly via an increase in resources available for research. With respect to this, the behaviour of a university that aims at maximizing prestige via research activities is very similar to the behaviour of a university that aims at maximizing enrolments. However, in the latter case we obtain e*=g+p, whereas in the former case a part of the budget available is allocated to research activities, such that e*<g+p. Moreover, maximizing r(e,q) implies a lower level of academic standards q* and a smaller number of students attracted s* compared to maximizing s(e,q).[16] However, as already emphasized above, in order to attract a positive number of students at all, a certain minimum level of educational efforts is needed. Consequently, the university will be able to squeeze out extra money for financing research activities only if total financial resources available per student exceed a lower bound $(g+p)^{min}$.

From the viewpoint of government, the message of the above results is obvious: For given governmental grants and tuition fees, a university that solely aims at maximizing prestige via research activities provides only a lower quality of education[17] for a smaller number of students compared to a university that solely aims at maximizing enrolments. The overall welfare effects of these different outcomes, however, depend on the weights assigned to education and research in the social welfare function which is beyond the scope of the present analysis.

4. Comparative Statics Analysis

Now, consider a marginal increase in governmental grants g, the effects of which are summarized in the second column of Table 1.[18] If the university under consideration solely aims at maximizing enrolments, we obtain de*/dg=1, dq*/dg>0 and ds*/dg>0.

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[16] To see this, note that in both cases the first order condition $s'_q = 0$ has to be satisfied. Hence, due to $s'_{qq} < 0$ (see [A.1] in Appendix I) and $s'_{qe} > 0$ (see [A.6] in Appendix II), a lower value of e* has always to be accompanied by a lower value of q and vice versa. Moreover, in both cases s(e,q) is maximized for given educational efforts per student. A lower value of e*, however, implies that s(e,q) attains its maximum at a lower absolute level.

[17] Here, “educational quality” refers to the chosen mix of educational efforts e and academic standards q. Formally, educational quality can be viewed as a function f(e,q) with $f'_e > 0$ and $f'_q > 0$.

[18] Concerning the derivation of the comparative statics results discussed in this Section see Appendix II.

Hence, in this case, increasing governmental grants leads to a higher educational quality and an increasing number of students attracted which, however, comes at the costs of a decreasing average ability. In contrast, if the university under consideration solely aims at maximizing prestige via research output, neither educational quality nor enrolments will change and the complete amount of extra money obtained from government will be absorbed by the research budget. Hence, in this case, there is no possibility to influence the university's behaviour via marginally varying governmental grants.

	dg>0	dp>0	−dg=dp>0	dz>0, dβ>0	dγ>0, $d\bar{R}$ >0
	Maximizing enrolments s(e,q)				
de*	1	1	0	0	0
dq*	+	+	0	0	0
ds*	+	?	−	+	−
	Maximizing the research budget r(e,q)				
de*	0	0	0	0	0
dq*	0	0	0	0	0
ds*	0	−	−	+	−
dr*	+	?	−	+	−

Table 1: Results of Comparative Statics Analysis.

Next, consider a marginal increase in the tuition fee p, the effects of which are summarized in the third column of Table 1. Let us begin again with a university that solely aims at maximizing enrolments. In this case we again obtain de*/dp=1 and dq*/dp>0. Consequently, marginally increasing the tuition fee has the same impact on educational efforts and academic standards as marginally increasing governmental grants. However, one important difference remains. Whereas the latter measure increases educational quality without increasing the students' costs, this is not true for the former one. Hence, whereas marginally increasing governmental grants always leads to an increasing number of students, the respective effect caused by marginally increasing the tuition fee is ambiguous. As shown in Appendix II, we obtain:

[12] $$ds^* = \frac{n[(\gamma\bar{R} + p - \beta)w'_e - w(e,q)]}{(1-\gamma)z\bar{R}qw(e,q)^2}dp\,.$$

The reason for this ambiguity is straightforward: According to [2], increasing the tuition fee by dp>0 has two opposite effects on the students' expected lifetime income. On the one hand, it directly decreases $E(R^j)$ by dp. On the other hand, due to de*=dp it increases the probability to become graduated by $w'_e dp$ which leads to an increase in expected lifetime income by $(1-\gamma)zq\alpha^j w'_e \overline{R}dp$. Hence, the total effect is given by $dE(R^j) = [(1-\gamma)zq\alpha^j w'_e \overline{R} - 1]dp$. Now, consider the student "at the margin" with ability $\alpha^j = \overline{\alpha}$. Inserting $\overline{\alpha}$ as given by [3] into $dE(R^j)$ yields:

[13] $$dE(R^j) = \frac{(\gamma\overline{R} + p - \beta)w'_e - w(e,q)}{w(e,q)} dp.$$

Comparing [12] and [13] reveals that ds*/dp>0 requires $dE(R^j)/dp > 0$ for the student at the margin. Due to e*=p+g and $w''_{ee} < 0$, whether or not the condition $dE(R^j)/dp > 0$ is satisfied depends on the initial magnitude of e. Starting with a comparatively low e, educational efforts exhibit a high "marginal productivity" in the sense of increasing the probability to become graduated. Hence, in this case the indirect effect of increasing $dE(R^j)$ by $(1-\gamma)zq\alpha^j w'_e \overline{R}dp$ will dominate such that $dE(R^j)/dp > 0$ and therefore ds*/dp>0. However, in a situation where educational efforts are already comparatively high, the direct effect of decreasing $E(R^j)$ will dominate such that $dE(R^j)/dp < 0$ and therefore ds*/dp<0. Consequently, in the case of maximizing enrolments, marginally increasing the tuition fee p will lead to an increasing enrolments only if total financial resources available per student, g+p, initially are comparatively low.

If the tuition fee p is marginally increased and the university under consideration solely aims at maximizing prestige via research output, we again obtain the result that neither educational efforts nor academic standards will change and the complete amount of extra money obtained from students will be absorbed by the research budget. However, in contrast to the case of increasing governmental grants, an increase in the tuition fee will lead to decreasing enrolments. As a consequence, the overall effect on the research budget is ambiguous (see Appendix II):

[14] $$dr^* = n\left[1 - \frac{\gamma\overline{R} - \beta + g + 2p - e}{(1-\gamma)z\overline{R}qw(e,q)}\right]dp.$$

Similar, to the above case, dr*/dp>0 requires that the direct effect of increasing the tuition fee dominates the indirect effect of decreasing enrolments. [19]

[19] Note, that from [12] and [14], respectively, we could easily derive conditions describing the optimal tuition fee from the viewpoint of the university. In this case, where the university is empowered to deliberately fix p, however, the second order conditions turn out to be highly complex.

From the viewpoint of government, the most interesting question might be, what happens if governmental grants are partially substituted by tuition fees, i.e. $-dg=dp>0$. As indicated by the results summarized in the forth column of Table 1, irrespective of the objectives pursued by the university, such a substitution would change neither educational efforts nor academic standards, but it would inevitably lead to decreasing enrolments and a decreasing research budget. Of course, as a positive side-effect, the average ability of the remaining population of students would increase. But nevertheless, the above results clearly indicate that partially substituting governmental grants by tuition fees can never be in the interest of the university.

Finally, assume a marginal increase in the other exogenous parameters z, β, γ and $\overline{R}$. As indicated by the results summarized in the last two columns of Table 1, irrespective of the objectives pursued by the university, there will be no change in educational efforts or academic standards. As a consequence, enrolments and the research budget will increase if studying becomes c.p. more attractive due to increasing leisure benefits or an increasing difference in income between graduates and non-graduates. Similarly, enrolments and the research budget will decrease if studying becomes c.p. less attractive due to increasing opportunity costs caused by an increase in γ or $\overline{R}$.

5. Summary and Prospects for Future Research

The present paper has developed a formal model describing the behaviour of a university within an equilibrium framework where demand for enrolments is traced back to the decision of potential students which aim at maximizing expected lifetime income. The key factors of this decision process are the students' preferences and abilities, the quality of education offered by the university and several external determinants like, e.g., tuition fees and the difference in income between graduates and non-graduates. In turn, the behaviour of the university in terms of educational efforts and the strength of academic standards depends on the demand for enrolments, on the amount of financial resources available from governmental grants or tuition fees and on the specific objectives pursued by the university.

Within this framework, the paper shows that for given financial resources a university that aims at maximizing prestige via research activities always provides only a lower quality of education for a smaller number of students compared to a university that aims at maximizing enrolments. Moreover, the paper reveals that the effects caused by changes in governmental grants or tuition fees, respectively, are quite different depending on the objectives pursued by the university. Yet, there is also one common feature: Irrespective of which utility function is maximized, partially substituting governmental grants by tuition fees would change neither educational efforts nor academic standards, but it would inevitably lead to decreasing enrolments. As a positive side-effect, however, the quality of the remaining population of students in the sense of their average ability would increase.

Of course, the above analysis still suffers from several simplifications. The most important one might be the absence of competition between different universities. In order to incorporate this issue, one could imagine an extended version of the above model with two regions and one university in each of them. In such an extended model, potential students in each region would have to decide between three alternatives: working within the non-academic sector, studying at the "home university" or studying at the university in the other region. The latter alternative would give rise to some kind of "mobility costs" and it has to be expected that the magnitude of these costs would be one of the key factors explaining the degree of competition between universities.

A second promising line of future research relates to welfare considerations. The model developed so far concentrates on the level of positive economic analysis. Hence, it only tries to describe the actual behaviour of universities, but it does not ask how universities should behave from the viewpoint of maximizing social welfare. In order to account for this shortcoming, one could imagine another extension of the above model where the government aims at fixing governmental grants and tuition fees in such a way as to maximize a social welfare function comprised of, e.g., the number of students which finally become graduated and the quantity of research output produced by the university.

Appendix I: Second Order Conditions

In the case of maximizing s(e,q), the second order condition requires $s''_{qq} < 0$. Twice differentiating [4] and accounting for the first order condition $w(e,q) = -qw'_q$ proofs:

$$\text{[A.1]} \quad s''_{qq} = \frac{n(\gamma\overline{R} + p - \beta)[2w'_q + qw''_{qq}]}{(1-\gamma)z\overline{R}q^4 {w'_q}^2} < 0.$$

In the case of maximizing r(e,q), the second order conditions require $r''_{ee} < 0$ and $r''_{ee}r''_{qq} - r''_{eq}r''_{qe} > 0$. Let us first consider $r''_{ee} < 0$. Twice differentiating $r(e,q) = (g + p - e)s(e,q)$ with respect to e yields $r''_{ee} = (g + p - e)s''_{ee} - 2s'_e$. Moreover, from differentiating [4] we obtain:

$$\text{[A.2]} \quad s'_e = \frac{n(\gamma\overline{R} + p - \beta)w'_e}{(1-\gamma)z\overline{R}qw(e,q)^2} > 0,$$

$$\text{[A.3]} \quad s''_{ee} = \frac{n(\gamma\overline{R} + p - \beta)[w(e,q)w''_e - 2{w'_e}^2]}{(1-\gamma)z\overline{R}qw(e,q)^3} < 0.$$

Consequently, the second order condition $r''_{ee} < 0$ is always satisfied. Now, let us turn to the second order condition $r''_{ee}r''_{qq} - r''_{eq}r''_{qe} > 0$. Using again $r(e,q) = (g + p - e)s(e,q)$ and accounting for $s'_q = 0$, the condition $r''_{ee}r''_{qq} - r''_{eq}r''_{qe} > 0$ can be re-calculated as:

$$\text{[A.4]} \quad (g + p - e)\{s''_{qq}[(g + p - e)s''_{ee} - 2s'_e] - (g + p - e)^2 {s''_{qe}}^2\} > 0.$$

First, assume the special case $w''_{qe} = 0$. Under this assumption, using [4] and accounting for the first order condition $w(e,q) = -qw'_q$ the second order condition [A.4] can be re-calculated as:

$$\text{[A.5]} \quad \frac{n^2(\gamma\overline{R} + p - \beta)^2}{(1-\gamma)^2\overline{R}^2q^8z^2{w'_q}^5}\Big\{w'_e[(g + p - e)w'_e(3w'_q + 2qw''_q) - 2qw'_q(2w'_q + qw''_{qq})] + (g + p - e)qw'_qw''_e[2w'_q + qw''_{qq}]\Big\} > 0$$

which holds due to $w'_e > 0$, $w''_{ee} < 0$, $w'_q < 0$, and $w''_{qq} < 0$. Hence, for $w''_{qe} = 0$ the second order condition $r''_{ee}r''_{qq} - r''_{eq}r''_{qe} > 0$ is always satisfied. For the more general case with $w''_{qe} \geq 0$ however, we explicitly have to assume $s''_{qq}[(g + p - e)s''_{ee} - 2s'_e] > (g + p - e)^2 {s''_{qe}}^2$ in order to guarantee $r''_{ee}r''_{qq} - r''_{eq}r''_{qe} > 0$.

Appendix II: Comparative Statics Analysis

Concerning the comparative statics of changes in governmental grants g it should be recognized that changing g has no *direct* effect on the number of students attracted, but only *indirect* effects via possible changes of e and q: $s(g)=s[e(g),q(g)]$. To obtain the results summarized in Table 1 for the case of maximizing enrolments, we employ the implicit function theorem to the first order condition $s'_q[q(g),e(g)]=0$. Due to de*/dg=1 this leads to $dq^*/dg=-s''_{qe}/s''_{qq}$. Calculating s''_{qe} from twice differentiating [4] and inserting the first order condition $w(e,q)=-qw'_q$ yields:

$$\text{[A.6]} \quad s''_{qe}=\frac{n(\gamma\bar{R}+p-\beta)[w'_e+qw''_{qe}]}{(1-\gamma)z\bar{R}q^4 {w'_q}^2}>0.$$

Now, from inserting [A.1] and [A.6] into $dq^*/dg=-s''_{qe}/s''_{qq}$ we obtain

$$\text{[A.7]} \quad \frac{dq^*}{dg}=-\frac{s''_{qe}}{s''_{qq}}=-\frac{w'_e+qw''_{qe}}{2w'_q+qw''_{qq}}>0$$

for the case of maximizing enrolments. Furthermore, the impact on enrolments can be calculated from the total derivative of $s[q(g),e(g)]$. Due to de*/dg=1 and $s'_q=0$ we obtain $ds^*/dg=s'_e$. Differentiating [4] with respect to e yields:

$$\text{[A.8]} \quad \frac{ds^*}{dg}=s'_e=\frac{n(\gamma\bar{R}+p-\beta)w'_e}{(1-\gamma)z\bar{R}qw(e,q)^2}>0.$$

For the case of maximizing the research budget, the effects of marginally increasing governmental grants can be calculated from differentiating $r=[g+p-e(g)]\cdot s[e(g),q(g)]$ with respect to g:

$$\text{[A.9]} \quad \frac{dr^*}{dg}=s(e,q)(1-e'_g)+(g+p-e)(s'_e e'_g+s'_q q'_g).$$

Accounting for the first order conditions $s'_q=0$ and $(g-e)s'_e=s(e,q)$, [A.9] can be reduced to $dr^*/dg=s(e,q)$. Hence, the amount of extra money obtained from government, $s(e,q)dg$, is completely absorbed by the research budget. As a consequence, we obtain de*/dg=dq*/dg=ds*/dg=0.

Now, consider a marginal increase in the tuition fee p. In contrast to the case considered above, it has to be recognized that changing p has not only indirect effects on enrolments via possible changes of e and q, but also a direct effect since it directly increases the costs of studying: $s(p)=s[e(p),q(p),p]$. Let us begin again with a university that solely aims at maximizing enrolments. Employing the implicit function theorem to the first order condition $s'_q[q(p),e(p),p]=0$ and accounting for de*/dp=1 yields $dq^*/dp=-(s''_{qe}+s''_{qp})/s''_{qq}$. Next, twice differentiating [4] with respect to q and p and accounting for $w(e,q)=-qw'_q$ reveals $s''_{qp}=0$. Hence, we obtain $dq^*/dp=-s''_{qe}/s''_{qq}$.

Comparing this with [A.7] shows that marginally increasing p has the same (positive) impact on q* as marginally increasing g. Moreover, due to $dq^*/dp = dq^*/dg$ we obtain dq*=de*=0 for the case of marginally substituting governmental grants by tuition fees, i.e., $-dg = dp > 0$.

The impact of marginally increasing p on the number of students attracted in the case of maximizing enrolments can be calculated from differentiating $s[q(p), e(p), p]$ with respect to p. Due to de/dp=1 and $s'_q = 0$ this leads to $ds^*/dp = s'_e + s'_p$. Here, s'_e indicates the indirect effect via increasing educational efforts and s'_p refers to the direct effect via increasing the costs of studying. Differentiating [4] with respect to e and p yields:

$$[\text{A.10}] \quad \frac{ds^*}{dp} = s'_e + s'_p = \frac{n[(\gamma\overline{R} + p - \beta)w'_e - w(e,q)]}{(1-\gamma)z\overline{R}qw(e,q)^2}.$$

In the case of maximizing the research budget, the effects of marginally increasing the tuition fee can be calculated from differentiating $r = [g + p - e(p)] \cdot s[e(p), q(p), p]$ with respect to p:

$$[\text{A.11}] \quad \frac{dr^*}{dp} = s(e,q)(1 - e'_p) + (g + p - e)[s'_e e'_p + s'_q q'_p + s'_p]$$

Accounting for $s'_q = 0$ and $(g + p - e)s'_e = s(e,q)$, [A.11] can be reduced to:

$$[\text{A.12}] \quad \frac{dr^*}{dp} = s(e,q) + (g + p - e)s'_p.$$

Since $s'_p < 0$ indicates only the direct effect on enrolments caused by increasing the tuition fee, this result implies, that educational efforts as well as educational demands will remain unchanged (de*/dp=dq*/dp=0). As a consequence, we obtain for the number of students attracted: ds*/dp=$s'_p < 0$. Moreover, using [4], dr*/dp can further be calculated as:

$$[\text{A.13}] \quad \frac{dr^*}{dp} = n\left[1 - \frac{\gamma\overline{R} - \beta + g + 2p - e}{(1-\gamma)z\overline{R}qw(e,q)}\right].$$

Finally, due to de*/dp=de*/dg=0 and dq*/dp=dq*/dg=0 we again obtain the result that marginally substituting governmental grants by tuition fees ($-dg = dp > 0$) has no impact on educational quality but leads to decreasing enrolments.

To obtain the comparative statics results concerning marginal variations of z, β, γ and $\overline{R}$ in the case of maximizing enrolments, apply the implicit function theorem to the first order condition $s'_q[q(\omega),e(\omega),\omega]=0$ with ω to be substituted by z, β, γ or $\overline{R}$, respectively. Accounting for de*/dω=0, this leads to $dq^*/d\omega = -s''_{q\omega}/s''_{qq}$. Next, twice differentiating [4] and accounting for $w(e,q)=-qw'_q$ reveals that the second partial derivatives $s''_{q\omega}$ evaluated at the optimum are given by $s''_{q\omega}=0$. Consequently, we obtain $dq^*/d\omega=0$ for $\omega\in[z,\beta\gamma,\overline{R}]$. Moreover, because e* and q* remain unchanged, the impact caused on enrolments, can easily be calculated from the first derivatives of [4]. This leads to ds*/dz>0, ds*/dβ>0, ds*/dγ<0 and ds*/d$\overline{R}$<0 as summarized in Table 1.

In the case of maximizing the research budget, the effects of marginally increasing z, β, γ or $\overline{R}$ can be calculated from differentiating $r=[g+p-e(\omega)]\cdot s[e(\omega),q(\omega),\omega]$ with respect to $\omega\in[z,\beta\gamma,\overline{R})$:

[A.14] $$\frac{dr^*}{d\omega} = -s(e,q)e'_\omega + (g+p-e)[s'_q q'_\omega + s'_e e'_\omega + s'_\omega].$$

Accounting for the first order conditions $s'_q=0$ and $(g+p-e)s'_e = s(e,q)$, [A.14] can be reduced to:

[A.15] $$\frac{dr^*}{d\omega} = (g+p-e)s'_\omega.$$

Since s'_ω indicates only the direct effect on enrolments caused by increasing ω, this result again implies that educational efforts as well as educational demands will remain constant: de*/dω=dq*/dω=0. As a consequence, the impact caused on enrolments and the research budget again can be calculated from the first derivatives of [4] with respect to z, β, γ and $\overline{R}$, respectively. This leads to the results shown in the last column of Table 1.

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