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A Spatially-related Note on Entrepreneurship and Economic Growth

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Abstract

A large and still growing body of literature suggests that entrepreneurship is of exceptional importance in explaining knowledge spillovers. Although quantifying the impact of entrepreneurial activity for economic growth is an interesting issue – particularly at the regional level – a concise formulation within a theoretical growth model is still missing. This paper in general tries to uncover the link between own- and neighbour-related regional entrepreneurial activity in innovation and regional growth within a spatial semi-endogenous growth model in the spirit of Jones (1995) reflecting recent empirical findings on entrepreneurial activity for economic growth. The paper derives an explicit solution for the transitional as well as for the balanced growth path level of ideas.

Keywords: entrepreneurship, economic growth, innovation, knowledge spillover
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1 Introduction

A recently past published strand of literature suggests that entrepreneurial activity and region's economic spatial patterns might be a potential source explaining knowledge spillovers endogenously. This is definitively a step forward to obtaining a more elaborated picture when explaining technological change endogenously.

The prevailing literature on endogenous growth assumes that technological progress is mainly driven by *R&D* investments leading to new ideas. In this context, the literature emphasizes in an explicit way the role of knowledge spillover which may increase the productivity in the *R&D* sector. Unfortunately, this aforementioned literature leaves out the spatial dimension of knowledge spillover, although a bulk of recent publications finds that knowledge spillover features a localized dimension rather than a globalized one (Acs and Varga (2002)).

Another strand of literature deals with the link of entrepreneurship and its impact on growth. It seems that research on entrepreneurship has mainly focused on the traits of an entrepreneur so far rather than the role of the entrepreneur as a person discovering and exploiting new knowledge (Shane and Venkataraman (2000)). Carre and Thurik (2003) argue that given entrepreneurs play a central role in explaining economic growth, this link should be addressed in future research, particularly when talking about the spatial dimension of this issue (Acs and Armington (2004)).

Acs and Varga (2005) and to a somewhat lesser extend, Varga and Schalk (2010) are the first to pick up the link between entrepreneurship, agglomeration and technological progress. Embedded in a cross section framework, Acs and Varga (2005) estimate the ideas' production function introduced by Jones (1995) and – controlling for agglomeration effects – they find that the effect of entrepreneurship on technological progress is highly significant. The drawback of their study is that they only take into account the *spillover effect* which may increase productivity but neglect a second source of spillover, mainly known

from quality-ladder growth framework proposed by Aghion and Howitt (1992): the *business-stealing effect*.

The focus of this paper is the link between entrepreneurship, agglomeration and technological progress in the presence of two distinct types generated by *R&D*: first the *spillover effect*, and second the *business-stealing effect*.

The contribution of the present paper is twofold. First, the paper develops a semi-endogenous growth model in the spirit of Jones (1995) that takes into account of the influence entrepreneurship and agglomeration on technological progress. To the best of my knowledge this is novel in the literature because the aforementioned studies are empirically focused. Second, the paper derives an explicit solution for the transitional as well as for the balanced growth path level of ideas.

The paper is structured as follows. Section 2 introduces the basic elements of the model. Section 3 derives its long run solution and discusses its implications. The analysis of the transitional dynamics is conducted in Section 4. Section 5 summarizes the main findings of the model, relates these to the recent empirical evidence and finally concludes.

2 The basic elements of the model

This section introduces the basic elements of the model. The paper considers a second generation growth model in the spirit of Jones (1995)¹. Although this model is partial, it is compatible with any general equilibrium framework that is in line with the ideas' production function proposed by Jones (1995) because the results we will derive in the following sections are independent of the wage rate w and the value of an idea V . Further labour is homogeneous and supposing a final good sector determines labour devoted to *R&D*.

¹For a discussion of different types of growth models and ideas' production functions as well as their specific appropriateness see e.g. Jones (1999, 2005).

2.1 *Spillover effect and business-stealing effect*

Assuming free entry into the *R&D* sector, we assume two channels of spillover which affect the development of new ideas. The first channel is the so-called *spillover effect* stemming from the fact that productivity growth will be affected due to entrepreneur's research activities as well as by the entrepreneur's own knowledge and knowledge-sources of other entrepreneurs. Hence, in general exploiting spillovers from other entrepreneurial activities in general results in less expensive innovations. Thus, there is a tendency for under-investment in *R&D* from a social perspective.

The second channel which I believe is worth elaborating is the *business-stealing effect* whereby innovations by a entrepreneur's competitor may decrease the entrepreneur's own market share. The contributions to the literature focusing on this second channel are relatively rare compared to the number of studies considering the first channel.² Going in line with Jones and Williamson (2000), we believe that it is important to discuss these two effects simultaneously for two reasons: first, empirical estimates of technological spillover effects tend to be over-estimated when neglecting the *business-stealing effect*. Second, most of the *R&D* subsidies schemes trying to resolve the market failure associated with the *spillover effect*, may be misspecified when it turns out that the *business-stealing effect* offsets or even dominates the *spillover effect*.

2.2 Geographical proximity

In contrast to the aforementioned strand of literature, we additionally assume that geographical proximity to the *R&D* source amplifies the *spillover effect*. As shown by Acs, Anselin and Varga (2002), Jaffe, Trajtenberg and Henderson (1993) and Varga (1998) for the US³ there is strong evidence that spillover are

²Aghion and Howitt (1992) and Jones and Williamson (2000) are some of the few exceptions which explicitly take into account the *business-stealing effect* in their theoretical work.

³Similar evidence is found for Europe. See for instance, Autant-Bernard (2001) and Fischer and Varga (2003).

bounded geographically. Focusing on the relationship between size of a region and the production of ideas, Varga (2000,2001) found a significantly positive relationship. Further it is believed that the countervailing *business-stealing effect* may be affected in a similar spatial way as the *spillover effect*.

2.3 Production of new ideas

In this way, the paper merges two strands of literature. On the one hand, the endogenous growth theory focusing on knowledge-spillovers and, on the other hand, the empirical focused literature dealing with entrepreneurship, agglomeration and endogenous growth.

Drawing these arguments together, the *R&D* sector develops new ideas according to:

$$\frac{\dot{A}}{A} = \left(\frac{1}{1 + \bar{\psi}} \right) \epsilon A^{\phi-1} L_R^{\lambda(\zeta, \delta, \beta)}, \quad (1)$$

where $L_R(t) = \zeta L(t)$, with $\zeta \in (0, 1)$ as the amount of labour which is employed in the *R&D* sector and \dot{A} is the derivative of the level of ideas A with respect to time t ⁴. ϕ , representing the *standing on shoulders effect*, and which directly associated with spillovers from codified knowledge⁵, is exogenous to the economy. $\epsilon > 0$ is a constant.

$\bar{\psi}$ reflects the severity of the explained *business-stealing effect*. Several items of equation (1) deserve to be discussed in more details. As argued before, we decompose this effect into a home and neighbouring *business-stealing effect*. Denoting $\bar{\psi}_h$ as the home *business-stealing effect* and accordingly, $\bar{\psi}_n$ as the corresponding neighbouring *business-stealing effect* weighted by $\delta \in (0, 1)$, we can explain $\bar{\psi}$ endogenously by:

$$\bar{\psi} = \nu(1 + \bar{\psi}_h)(1 + \bar{\psi}_n)^\delta, \quad (2)$$

⁴Time t is continuous. The fraction $(1 - \zeta)L(t)$ is employed in the final good sector.

⁵Refer to Polanyi (1976) for the distinction between the tacit and codified dimension of knowledge.

with $\nu \in [0, 1)$, $\bar{\psi}_n > 0$ and $\bar{\psi}_h > 0$. Following Varga (2001, 2001), the severity of δ on $\bar{\psi}$ depends positively on the size of neighbouring regions⁶.

Second, $\lambda(\cdot)$ captures the *spillover effect* which is also labeled as the *stepping on toes effect*. In other words, $\lambda(\cdot)$ reflects to which extent tacit knowledge spills over from the *R&D* sector. $\lambda(\cdot)$ is treated to be endogenous because we believe that the level of entrepreneurial activities as well as the spatial proximity to the source of *R&D* activities affects $\lambda(\cdot)$ in a positive way. This may be conjectured from the empirical evidence that particularly young entrepreneurial firms push technological innovations⁷. Audretsch and Keilbach (2004) use regional data for 8 OECD countries and find that up to 40% of economic growth is associated with firm turn over, whereas firm turn over is directly associated with entrepreneurship and young innovative firms because firm entry and exit rates are positively correlated⁸.

2.4 Specification of $\lambda(\cdot)$

The question which arises is how to specify $\lambda(\cdot)$ in order to catch, first, entrepreneurial activity and, second, the advantage of benefiting from the geographical proximity to the source of neighbouring entrepreneurial *R&D* activities. It is assumed that $\lambda(\cdot)$ may be approximated best by a real-valued and differentiable logistic function $f(\lambda(\cdot))$, associated with a non-negative, bell-shaped first order derivative, although other specifications may be possible. Nevertheless the logistic specification has been chosen because, first, the growth of $\lambda(\cdot)$ is limited by converging to a saturation level β . Second,

⁶Alternatively, we can interpret the decomposition of the *business-stealing effect* as a *within* and *across* industries *business-stealing effect*. This interpretation is in line with Li (2001).

⁷See Carree and Thurik (1998) and Audretsch and Thurik (2001) for instance. It has to be pointed out that several contributions find that fewer and larger firms push technological innovations. Regarding the last topic, please refer to Antony, Klarl and Maußner (2011) for a detailed literature review.

⁸As mentioned by Audretsch (2007), today's entrepreneurship results in tomorrow's SMEs.

for different levels of entrepreneurial activities and agglomeration we should expect a non-linear pattern for the growth of $\lambda(\cdot)$ from which it is unclear ex ante whether it may hamper or accelerate the growth of $\lambda(\cdot)$. Hence, the logistic specification in this sense is more flexible than the alternative exponential specification.

Entrepreneurial activity in our model is directly related to research intensity regarding *R&D* employment. Antony, Klarl and Maußner (2011) within a second generation growth model derived an optimal firm size distribution showing that without heterogeneous credit frictions young firms tend to employ more researchers than older firms instead of benefiting more from imitation than from innovation. Consequently, an increasing research intensity may increase $\lambda(\cdot)$.

$\lambda(\cdot)$ may be specified as

$$\lambda(\zeta, \delta, \beta) = \frac{\alpha}{1 + \text{Exp}[\beta - \kappa(1 + \Omega(\zeta, \delta))]}, \quad (3)$$

β stands for the region's specific institutional conditions, which may be reflected by costs of complying with regulations, bureaucratic restrictions, credit frictions and red tape starting a business as an entrepreneur. As we can see from equation (3), an increasing β discourages entrepreneurship. α , technically spoken, stands for the saturation level to which the sigmoid function converges. Following Jones (1995), we restrict $\alpha < 1$. κ is handled as a weighting parameter for $\Omega(\cdot)$.

Geographical proximity benefiting from the source of neighbouring entrepreneurial *R&D* activities is approximated by a weighting factor $\delta \in (0, 1)$. $\Omega(\cdot)$ – reflecting the entrepreneurial sector research intensity – is then given by $\Omega(\zeta, \delta) = \left(\frac{1}{1-\zeta}\right)^\delta$. $\Omega(\cdot)$ is increasing both in ζ and δ . Following Varga (2000, 2001), the severity of δ on $\Omega(\cdot)$ depends positively on the size of neighbouring regions.

It is worth to note, that the assumptions regarding the specification of $\lambda(\cdot)$ are in line with the empirical works of Acs and Varga (2005) and Varga and Schalk (2010). The next section concentrates on the growth along the balanced

growth path.

3 Growth along the balanced growth path

In this section, we derive an explicit solution for the balanced growth path level of the entrepreneurs' innovative activities. As in Jones (1995), along a balanced growth path, the growth rate of ideas $\frac{\dot{A}}{A} \equiv g$ is constant. This growth rate will be only constant if the denominator and nominator of the right-hand's side of equation (1) grow with the same rate. Taking log-derivatives of equation (1), and accounting for the fact that the time derivative of $\lambda(\cdot)$ is zero⁹, for the balanced growth rate g^* we arrive at

$$g^* = \frac{\lambda(\cdot)n}{1 - \phi}. \quad (4)$$

From the growth rate (4) we can also infer the level of ideas A^* on the balanced growth path (bgp). In general, we associate the bgp with constant growth rates of labor and ideas. Employing equation (1) together with equation (4) and solving for $A(t)$ in terms of the initial labour force $L(0)$, for the level of ideas on the bgp, A^* , we obtain

$$A^* = \left(\frac{\lambda(\zeta, \delta, \beta)n}{(1 - \phi)} \frac{(1 + \bar{\psi})}{\epsilon} \right)^{-\frac{1}{1-\phi}} (L(0) \text{Exp}[nt]\zeta)^{\frac{\lambda(\zeta, \delta, \beta)}{1-\phi}}. \quad (5)$$

From equation (5) we see that on the bgp, $A(t)$ is growing with the constant rate n as $L(t)$ does. What are the implications? First, from equation (4) we learn that only the *spillover effect* produces a long-run effect on the growth rate of ideas as well as on the level of ideas on the bgp. Second, the *business-stealing effect* affects the level of ideas on the gbp, but not the growth rate. With respect to the growth rate of ideas on the bgp, this implies that the *spillover effect* clearly dominates the *business-stealing effect* in the long

⁹This follows directly from the fact that $L(t)$ is growing with constant rate n .

run. Empirically, to show this is a challenging task. To the best of my knowledge, there is only one comprehensive study, that has focused on this issue. Bloom, Schankerman and Van Reenen (2007), using panel data on U.S. firms between 1980 and 2001, find that the *spillover effect* quantitatively dominates the *business-stealing effect* with respects to their impact on firm performance. Controlling for firm size, they find that this gap decreases with decreasing firm size but still remains positive.

In a similar way, we have to interpret the results obtained from Acs and Varga (2005). Ignoring the *business-stealing effect* but assuming that $\lambda(\cdot)$ is endogenously determined by agglomeration and entrepreneurship, they find that entrepreneurship and agglomeration significantly increases $\lambda(\cdot)$ and hence increases g^* . However, the obtained results and implications should be handled with care: The caveat of Acs and Varga (2005)'s study is that they ignore the effects of an increased $\lambda(\cdot)$ on the level of ideas, even on the bgp. This may lead to biased estimates. The best way, to make this point clear, is to discuss the transitional dynamics of our model.

4 Equilibrium during transition

We aim to characterize the equilibrium dynamics during a transition followed by a shock shifting the parameter δ – reflecting the aforementioned spatial dependence – towards the upper level $\tilde{\delta}$. Shifting δ implies shifting λ to $\tilde{\lambda}$, by increasing $\Omega(\cdot)$ to $\tilde{\Omega}(\cdot)$. Further $\bar{\psi}$ increases to $\tilde{\psi}$. Thus, shifting δ produces two opposing effects during transition: First, domestic firms may benefit from an increased knowledge spillover-pool stemming from extended research activities of neighbouring entrepreneurs, not only during transition, denoted by equation (4) but also in the long run, reflected by an increased long-run growth rate \tilde{g}^* . Second, by increasing δ , the *business-stealing effect* becomes c.p. more dominant as $\bar{\psi}$ increases to $\tilde{\psi}$.

In order to perform this analysis, the timing, as well as the type of the shock perturbing the economy, must be specified. To keep the analysis as

simple as possible, we assume that the shock is permanent and unannounced. Additionally, the shock will occur right at the beginning of the transition period in $t = 0$.

First, we aim to characterize the evolution of the level of ideas $A(t)$ over time.

Lemma 1. *For all $t \geq 0$, the time path of the ideas is represented by*

$$A(t) = \left\{ A^{*1-\phi} + \frac{\epsilon(1-\phi)(\zeta L(0))^{\tilde{\lambda}}(\text{Exp}[nt\tilde{\lambda}(\cdot)] - 1)}{(1+\tilde{\psi})n\tilde{\lambda}(\cdot)} \right\}^{\frac{1}{1-\phi}}. \quad (6)$$

Proof. The solution to the homogeneous first-order differential equation (1) may be known to be given by exploiting the fact that $\frac{dA(t)}{dt} \equiv \dot{A}(t)$. Rearranging terms on the left hand's side as well as on the right hand's side of $\frac{dA(t)}{dt} \equiv \dot{A}(t)$ and by employing equation (1) we find $\int \frac{dA(t)}{A(t)^\phi} = \int \frac{1}{1+\tilde{\psi}} \epsilon L(t)^{\tilde{\lambda}(\cdot)} dt$. Now, both sides can be integrated separately. By using $A(0) = A^*$ as an initial condition, we arrive at equation (6). \square

Consulting Lemma 1, we observe that the stock of ideas increases during transition in its two arguments $A(t)$ and $L(t)$ for given starting values $A(0) = A^* > 0$ and $L(0) > 0$. Further and worth to note is that equation (6) is directly influenced by $\tilde{\lambda}$ and $\tilde{\psi}$.

The next lemma will help us to analyze the transition of the growth rate of ideas.

Lemma 2. *For all $t \geq 0$, the time path of the growth rate of ideas $g(t)$ is given by*

$$g(t) \equiv \frac{\dot{A}(t)}{A(t)} = \left(\frac{1}{1-\phi} \right) \left(\frac{1}{A(t)^{1-\phi}} \right) \left(A^{*1-\phi} \tilde{\lambda}(\cdot) n + \frac{(\text{Exp}[nt\tilde{\lambda}(\cdot)] - 1) \epsilon (1-\phi)}{(1+\tilde{\psi})} \right). \quad (7)$$

Proof. The derivative of equation (6) with respect to time t is given by $\dot{A}(t) = \left(\frac{1}{1-\phi} \right) A(t)^\phi \left(A^{*1-\phi} \tilde{\lambda}(\cdot) n + \frac{(\text{Exp}[nt\tilde{\lambda}(\cdot)] - 1) \epsilon}{(1+\tilde{\psi})} \right)$. Dividing the last derived

expression by $A(t)$ results in equation (7). \square

The question in front is whether the transition of $g(t)$ given by equation (7) is conditioned on a certain set of starting-values or not. To tackle this issue, it may be helpful decomposing the growth rate of ideas $g(t)$ into an *idea-level effect* denoted by $B(t) \equiv \frac{1}{A(t)^{1-\phi}} (A^{*1-\phi} g^*)$ and a *research-intensity effect* defined as $C(t) \equiv \frac{1}{A(t)^{1-\phi}} \left(\frac{(\text{Exp}[nt]\zeta L(0))^{\lambda(\cdot)\epsilon}}{(1+\psi)} \right)$. Hence, $g(t) = B(t) + C(t)$ with $\dot{g}(t) = \frac{\partial B(t)}{\partial t} + \frac{\partial C(t)}{\partial t} \equiv b(t) + c(t)$.

Lemma 3. For $t \rightarrow \infty$, (1) $B(t)$ converges to zero and (2), $C(t)$ converges to \tilde{g}^* .

Proof of Lemma 3 (1) Immediate from Lemma 1: $A^* < A(0)$ and $A(t)$ is growing faster than A^* off the bgp. Hence, $\left(\frac{A^*}{A(t)}\right)^{1-\phi} \rightarrow 0$ for $t \rightarrow \infty$.

(2) From $\lim_{t \rightarrow \infty} \{C(t)\} = \lim_{t \rightarrow \infty} \left\{ \frac{1}{A(t)^{1-\phi}} \left(\frac{(\text{Exp}[nt]\zeta L(0))^{\lambda(\cdot)\epsilon}}{(1+\psi)} \right) \right\}$ we find by applying L'Hôpital's rule that $\lim_{t \rightarrow \infty} \{C(t)\} = \tilde{g}^*$. \square

In words, Lemma 3 suggests that $g(t)$ converges to \tilde{g}^* in $[0, \infty)$. However, the exact transition pattern of $g(t)$ is entirely governed by $B(t)$ and $C(t)$ and Lemma 3 clearly does not provide sufficient information answering this question. Hence, making any conclusions regarding the exact pattern of transition of $g(t)$ requires a more elaborate analysis of $B(t)$ and $C(t)$ during transition.

First, let us neglect the negative *business-stealing effect*. Then $B(t)$ converges to zero, because the development of new ideas by the entrepreneurs' home and abroad research activities increase the stock of ideas $A(t)$ and thus $\left(\frac{A^*}{A(t)}\right)^{1-\phi}$ decreases monotonically by a steadily increasing level of ideas. It should be clear that the negative *business-stealing effect* accelerates the convergence of $\left(\frac{A^*}{A(t)}\right)^{1-\phi}$ to zero, given it is sufficiently small.

Second, by a given population $L(t)$, the ratio $\frac{L(t)}{A(t)}$ may jump to a lower level as $\lambda(\cdot)$ increases. Hence, a given number of researcher produces an increased number of ideas because of benefiting from knowledge-spillover stemming from neighbouring research activities. Hence, the actual growth rate of ideas $g(t)$

is higher than the long-run growth rate \tilde{g}^* and thus the research intensity $\frac{L(t)}{A(t)}$ gradually declines¹⁰ with the growth rate of ideas until the economy converges to its steady state \tilde{g}^* . Although the negative *business-stealing effect* has not any impact on the long run growth rate \tilde{g}^* , it indeed affects the convergence speed of $C(t)$ to its steady state.

Although not in the direct scope of this contribution, we may conjecture that a hump-shaped pattern of $g(t)$ may only occur if there will be a change in dominance from the negative *idea-level-effect* to the positive *research-intensity-effect* for a given *business-stealing effect*. Otherwise, if the positive *research-intensity effect* dominates the negative *idea-level effect* during transition, we should expect a positively monotonically transition from g^* to \tilde{g}^* ¹¹. The next Proposition extracts an initial condition under which $g(t)$ exhibits a hump-shape pattern during transition.

Proposition 1. *The growth rate of ideas may exhibit a hump-shaped pattern, provided that $g(0) > \sqrt{\tilde{g}^*C(0) + g^*B(0)}$ holds.*

Proof. See Appendix A.4. □

In words, from the (Proof of) Proposition 1 we can learn, first, that $-b(t)$ and $c(t)$ cross for sure at, given the existence condition $g(0) > \sqrt{\tilde{g}^*C(0) + g^*B(0)}$ is met. Second, the curves cross at $t = \tilde{t}$, and, third, \tilde{t} is unique.

The next Proposition states that we can exclude any hump-shaped pattern of the growth rate of ideas $g(t)$, provided that $c(t) \geq -b(t)$ for all t in $[0; \infty)$. Then, as we will see, the evolution of $g(t)$, as well as the convergence to \tilde{g}^* , is governed entirely by the evolution of $c(t)$.

Proposition 2. *The growth rate of ideas increases strictly monotonically and converges to \tilde{g}^* , provided that $g(0) < \sqrt{\tilde{g}^*C(0) + g^*B(0)}$ holds.*

¹⁰The growth rate of $C(t)$ during transition is given by $g_{C(t)} = \tilde{\lambda}n + (\phi - 1)g(t)$. Now $g_C(t) < 0$ given $\tilde{g}^* < g(t) \forall t$ et vice versa.

¹¹Please remember that only a change from the negative *idea-level effect* to the positive *research-intensity effect* during transition is possible, because $B(t)$ decreases monotonically to zero with $t \rightarrow \infty$, whereas $C(t)$ converges to $\tilde{g}^* > 0$ with $t \rightarrow \infty$.

Proof. See Appendix A.5. □

To sum up, Proposition 2 suggests, that for $g(0) < \sqrt{\tilde{g}^*C(0) + g^*B(0)}$, $g(t)$ exhibits a monotone convergence towards its steady state¹². This convergence appears to be only positive in the sense that $g(t)$ increases strictly monotonically to its steady state denoted by $\tilde{g}^* > g(0)$. The important insight we obtain from conducting the transitional dynamics analysis is that, first, the *business-stealing effect* is only effective during transition but, second, even in the long run, the *spillover effect* leads to a higher level of ideas.

5 Summary

5.1 Discussion of theoretical results

Varga and Schalk (2010) and Acs and Varga (2005) endogenizes entrepreneurial activity and agglomeration effects on knowledge spillovers based on a second generation growth model. However, conducting their empirical analysis, they concentrated on growth rates of ideas alone not taking into account the level effects induced by equation (5). From our model we may conjecture that focusing solely on growth rates is not sufficient, even if we would argue that the *business-stealing effect* is not from relevance.

Our model predicts that the *business-stealing effect* is only a transitional phenomenon (see equation (7)), whereas the *spillover effect* affects growth rates on as well as off the bgp. This is particularly important if the empirical analysis is based on long time series. Obviously, ignoring this issues, estimates could be biased.

5.2 Conclusion

The recent literature on entrepreneurship suggests that entrepreneurs might play a decisive role in terms of exploiting new technological opportunities. If

¹²We have skipped the possibility of $g(0) = \sqrt{\tilde{g}^*C(0) + g^*B(0)}$ as a non-relevant case.

so, the impact of entrepreneurship explaining growth defines an important research question, particularly at the spatial level. Empirical evidence find that the effect of entrepreneurship contributes significantly to technological change.

This paper picks up this research question and aims to uncover the link between own- and neighbour-related regional entrepreneurial activity in innovation and regional growth within a spatial semi-endogenous growth model in the spirit of Jones (1995) reflecting recent empirical findings on entrepreneurial activity for economic growth. The paper derives an explicit solution for the transitional as well as for the balanced growth path level of ideas reflecting entrepreneurs' innovative activities. In contrast to the empirical studies mentioned in this paper, the model accounts for the fact that *R&D* generates at least two distinct effects: a *spillover effect* and a *business-stealing effect*.

The main findings are: (1) entrepreneurial activity explaining the *spillover effect* endogenously, has a positive impact on the growth rate of ideas as well as on the level of ideas on the bpg. Hence, the paper matches the empirical findings in the long run. (2) Further, our model predicts that the negative *business-stealing effect* is a transitional phenomenon which may influence the growth rate of ideas off the bpg but even has effects on as well as off the bpg for the level of ideas. The paper concludes that empirical studies so far ignore this negative *business-stealing effect*. (3) Even if we would argue that this effect may be neglected in the light of the empirical challenge to identify this effect from the data and hence exclusively draws attention to the *spillover effect* as predicted by Varga and Schalk (2010) and Acs and Varga (2005), there is another source of possible biased estimates: the ignorance of the level effects of ideas on the bpg as predicted by our model. (4) Furthermore, by studying the transitional dynamics of our model, we obtain a deeper understanding of what exactly drives the dynamic of the growth rate of ideas. We have identified a condition separating a hump-shaped evolution of the growth rate of ideas from a monotone convergence to its long-run level. A comprehensive interpretation

of this is clearly beyond the direct scope of this contribution but provides an avenue for further research.

Appendix

Appendix A.1

Lemma 4 $-b(0) > 0$ irrespective the sign of $\dot{g}(0)$.

Proof. Immediate from $-b(0) = (1 - \phi)B(0)[g(0) - g^*]$ and $g(0) = B(0) + C(0)$ with $C(0) > 0$ and $B(0) = g^* > 0$. \square

Appendix A.2

Lemma 5 $\dot{c}(0) > 0$ for $\dot{g}(0) < 0$ and $\dot{c}(0) \geq 0$ for $\dot{g}(0) > 0$.

Proof. First, let $\dot{g}(0) < 0$ with $c(0) < -b(0)$. Then it is trivial to show that $\dot{c}(0) > 0$ because $(1 - \phi)(\tilde{g}^* - g(0))^2 > \dot{g}(0)$ with $\phi \in (0, 1)$. Second, let $\dot{g}(0) > 0$ with $c(0) > -b(0)$. Now, provided that $c(0) \in (-b(0), (1 - \phi)(\tilde{g}^* - g(0))^2 - b(0)]$ for all valid sets of starting values $\{-b(0), g(0)\}$, it is guaranteed that $\dot{c}(0) > 0$. Regarding the lower bound, we have $c(0) > -b(0)$. Hence, what remains to be shown is that for the upper bound

$$c(0) \leq (1 - \phi)(\tilde{g}^* - g(0))^2 - b(0) \quad (8)$$

may hold for all valid sets of starting values $\{-b(0), g(0)\}$. This may be proofed in the following way: We know that for t^{max} we have

$$g(t^{max}) = -\sqrt{\frac{\dot{g}(t^{max})}{1 - \phi}} + \tilde{g}^*, \quad (9)$$

provided $\dot{g}(0) > 0$. Now, assuming that $t^{max} = t = 0$ without loss of generality and inserting equation (9) in equation (8) we find that $c(0) = c(t^{max})$. Hence, $c(0)$ may never exceed $(1 - \phi)(\tilde{g}^* - g(0))^2 - b(0)$ and therefore $\dot{c}(0) > 0$ for $\dot{g}(0) > 0$ for $\forall t < t^{max}$. \square

Appendix A.3

Lemma 6 $-\dot{b}(0) < 0$ given $\dot{g}(0) < 0$ and $-\dot{b}(0) \leq 0$ given $\dot{g}(0) > 0$.

Proof. The first part is easy to verify. Let $\dot{g}(0) < 0$, we arrive at $c(0) < -b(0)$.

Hence, $-\dot{b}(0) = (1 - \phi)[b(0)(g(0) - g^*)^2 + B(0)\dot{g}(0)] < 0$ due to $b(0) < 0$. For the second part, let $\dot{g}(0) > 0$. Now $c(0) > b(0)$. Employing again the fact that $-\dot{b}(0) = (1 - \phi)[b(0)(g(0) - g^*)^2 + B(0)\dot{g}(0)]$, we arrive at $\frac{-b(0)}{g^*}g(0) \stackrel{\leq}{\geq} c(0)$ or after some rearrangements we obtain

$$\underbrace{-b(0)}_{\in(0,1)} \underbrace{\left(1 + \frac{C(0)}{g^*}\right)}_{>1} \stackrel{\leq}{\geq} \underbrace{c(0)}_{\in(0,1)} \Rightarrow -\dot{b}(0) \stackrel{\leq}{\geq} 0 \text{ for } \dot{g}(0) > 0. \quad (10)$$

Obviously, $\left(1 + \frac{C(0)}{g^*}\right)$ must be sufficiently close to one and/or $-b(0)$ must be close to zero meeting equation (10). \square

Appendix A.4

Proof of Proposition 1. For $g(0) > \sqrt{\tilde{g}^*C(0) + g^*B(0)}$, we have $\dot{g} < 0$. Accordingly this implies, that $-b(0) > c(0)$. Note that the partial derivatives of $-b(0)$ and $c(0)$ wrt time are given by $-\dot{b}(0)$ and $\dot{c}(0)$. Evaluating $-\dot{b}(0)$ and $\dot{c}(0)$ further, we find that $-\dot{b}(0) = (1 - \phi)g^*(g(0) - g^*) = (1 - \phi)g^*(C(0)) > 0$. Depending on the sign of $\tilde{g}^* - g(0)$, which may be positive or negative, $c(0) = (1 - \phi)C(0)(\tilde{g}^* - g(0))$ may be positive or negative. Next, recall that $-b(0) > c(0)$. For this we find that $-\dot{b}(0)$ is strictly decreasing since $-\dot{b}(0) = (1 - \phi)[g^*b(t)(g(0) - g^*) + B(0)\dot{g}]$ because $\dot{g}(t) < 0$ and $b(0) < 0$. From Lemma 4¹³ we know that $-b(0)$ is decreasing. Additionally, we know that $\dot{c}(0) = (1 - \phi)[c(0)(\tilde{g}^* - g(0)) - C(0)\dot{g}(t)]$ is strictly increasing because $(1 - \phi)(\tilde{g}^* - g(0))^2 > \dot{g}(t)$. Now let $t > 0$ and let $\dot{g}(0) < 0$. Then $-b(t)$ decreases monotonically from $-b(0)$ to $-b(\tilde{t})$ and $c(t)$ increases monotonically from $c(0)$ to $c(\tilde{t})$, where they meet each other. Basically, at $t = \tilde{t}$ we have that $-b(\tilde{t}) = c(\tilde{t})$ which states that $\dot{g} = 0$. Now it is important to note that for $\dot{g}(\tilde{t}) = 0$, $g(\tilde{t})$ obviously does not fulfill the steady state condition $g(\tilde{t}) = \tilde{g}^*$. This can be seen straightforwardly as follows: first, at $g(\tilde{t})$ we know that

¹³See Appendix A.1.

$g(\tilde{t}) = \sqrt{\tilde{g}^*C(0) + g^*B(0)}$ must hold. Next, rewrite $g(\tilde{t}) = \sqrt{\tilde{g}^*C(0) + g^*B(0)}$ as a weighted sum of the steady state growth rates g^* and \tilde{g}^* :

$$g(\tilde{t}) = \xi(\tilde{t})\tilde{g}^* + (1 - \xi(\tilde{t}))g^* \quad (11)$$

with $\xi(\tilde{t}) \equiv \frac{C(\tilde{t})}{B(\tilde{t})+C(\tilde{t})}$. Inserting equation (11) in $-\dot{b}(\tilde{t})$ and $\dot{c}(\tilde{t})$ we arrive at $-\dot{b}(\tilde{t}) = (1-\phi)b(\tilde{t})\xi(\tilde{t})(\tilde{g}^* - g^*) < 0$ and $\dot{c}(\tilde{t}) = (1-\phi)c(\tilde{t})(1-\xi(\tilde{t}))(\tilde{g}^* - g^*) > 0$. Moreover, $c(t)$ increases monotonically to $c(t^{cmax})$ in $[\tilde{t}, t^{cmax})$, whereas $-b(t)$ decreases monotonically to $-b(t^{cmax})$ in $[\tilde{t}, t^{cmax})$, which implies that $\dot{g}(t)$ turns its sign to $\dot{g}(t) > 0$ in $(\tilde{t}, t^{cmax}]$. This can be proofed in the following way: At $t = t^{cmax}$ we have that $c(t^{cmax}) = -b(t^{cmax}) + (1-\phi)(\tilde{g}^* - g(t^{cmax}))^2$ which immediately implies that $c(t^{cmax}) > -b(t^{cmax})$. Thus, $\dot{g}(t^{cmax})$ must be positive and increasing in (\tilde{t}, t^{cmax}) ¹⁴. Now, for $t^{cmax} \mapsto \infty$, $c(t)$ decreases and converges asymptotically to $-b(t)$, because $(\tilde{g}^* - g(\tilde{t}))^2$ converges to 0 as $g(\tilde{t})$ converges to \tilde{g}^* . This implies that $\dot{g}(t)$ exhibits a turning point at $t = t^{cmax}$, because $\dot{g}(t)$ is still positive but now decreasing in (t^{cmax}, ∞) . Hence, for $t^{cmax} \mapsto \infty$ we conclude that $c(\infty) = -b(\infty) = 0$ with $\dot{g}(\infty) = 0$ and hence $g(\infty) = \tilde{g}^*$. \square

Appendix A.5

Proof of Proposition 2. Basically, the proof of Proposition 2 is closely related to the proof of Proposition 1. From Proposition 1 we know that $\dot{g}(t) > 0$ which implies that $-b(0) < c(0)$ and, hence, $\tilde{g}^* - g(0) > 0$. Now taking the time derivative of $-b(0) < c(0)$ we arrive at $-\dot{b}(0) < \dot{c}(0)$.

From Lemma 5¹⁵ we know that $\dot{c}(0) > 0$, attaining a maximum at $t = t^{cmax}$ with $c(t^{cmax}) = -b(t^{cmax}) + (1-\phi)(\tilde{g}^* - g(t^{cmax}))^2$. Hence, $c(t^{cmax}) > -b(t^{cmax})$. Additionally, Lemma 6¹⁶ suggests that $-\dot{b}(0)$ is now either increasing or de-

¹⁴If we rewrite $c(t^{cmax}) = -b(t^{cmax}) + (1-\phi)(\tilde{g}^* - g(t^{cmax}))^2$ as $g(t^{cmax}) = \tilde{g}^* - \frac{\sqrt{\dot{g}(t^{cmax})}}{1-\phi}$, we can conclude, first, that $\dot{g}(t^{cmax})$ must be positive and, second, that \tilde{t} must be smaller than t^{cmax} .

¹⁵See Appendix A.2.

¹⁶See Appendix A.3.

creasing. Provided that $-\dot{b}(0) > 0$, at $t = t^{bmax}$ we have $c(t^{bmax}) = -b(t^{bmax}) + (1 - \phi)(g(t^{bmax}) - g^*)^2$ which clearly states that $c(t^{bmax}) > -b(t^{bmax})$. It may be helpful consulting the Proof of Proposition 1¹⁷ for completing the Proof of Proposition 2. For $t^{cmax} \mapsto \infty$, $c(t)$ converges monotonically to zero, as does $b(t)$. Because of the fact that $(1 - \phi)(\tilde{g}^* - g(t^{bmax}))^2$ is strictly positive, $c(t)$ cannot be smaller than $-b(t)$ during the convergence process in $[t^{cmax}; \infty)$. Thus, $c(t) \geq -b(t)$ for all t in $[0; \infty)$. Otherwise, for $-\dot{b}(0) < 0$ decreases monotonically for all t in $[0; \infty)$ from $c(0) > -b(0) > 0$ to zero. \square

¹⁷See Appendix A.4.

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